DETERMINATION OF THE TRANSFER FUNCTION OF A NONLINEAR CIRCUIT FOR PREDICTION OF INTERMODULATION CHARACTERISTICS

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Summary

This paper describes a frequency domain analysis technique developed to determine the transfer function of a nonlinear circuit from the harmonic outputs generated when the circuit is driven at specific frequencies by a single, large-signal sinusoid. The technique can be used to analyze power amplifiers and mixers for gain compression, intermodulation distortion, AM to PM conversion, etc. Examples are provided to illustrate the validity and accuracy of this technique.

Introduction

Traditional methods for analyzing the outputs of nonlinear circuits and devices, such as amplifiers and mixers, have generally relied on various time domain techniques. Such methods have the computational disadvantage that they must run until a steady-state is reached and then continue for a time period given by the lowest frequency present in the circuit. This requires considerable computation time when long time constants are present and the excitation signals are close in frequency, i.e. a low difference frequency is produced.

This paper introduces a technique that generates a transfer function for a nonlinear circuit that is valid over a specified input signal amplitude and frequency range (1,2). A principal advantage of obtaining a transfer function for a circuit is that it allows for multiple circuit analyses to be performed quickly and efficiently, e.g. those required to determine gain compression, third order intercept, AM to PM conversion, etc. for an amplifier. A second advantage of this technique is that because the analysis needed to generate the transfer function is performed in the frequency domain, it is not subject to the computational problems and limitations inherent in a time domain analysis.

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Technique Description

The basic concept behind this frequency domain analysis technique is that the input-output relationship in a circuit can be modelled by an equivalent representation where the circuit output consists of a summation of individual differential order outputs. For example, a circuit in which the circuit input is a voltage signal **V** and the circuit output is a current signal **I**, would be represented by the equivalent circuit shown in Figure 1.

The output **I** is the summation of the individual differential order currents $\mathbf{I}_i$. This is given in equation form as

$$I = \sum_{i=-\infty}^{\infty} I_i \quad (1)$$

where the $\mathbf{I}_{-1}$ term would correspond to inductive effects, the $\mathbf{I}_0$ term to resistive effects, the $\mathbf{I}_1$ term to capacitive effects, all the other negative order terms to multi-integral effects, and all the other positive order terms to multi-differential effects.

The output generated by each differential order "block" in Figure 1 is represented by a transfer function relating the circuit output to the circuit input. This transfer function is of the polynomial form (for the example shown in Figure 1)

$$I_i = K_{i\phi} + [K_{i1} + K_{i2}V + \dots + K_{in}V^{n-1}] \frac{\partial^i V}{\partial t^i} \quad (2)$$

where $\partial^i \mathbf{V}/\partial t^i$ represents the i^{th} time derivative of the input voltage **V**, K_{ij} (j=0,n) are constants consisting of real and imaginary components, and n is a positive integer that represents the polynomial order deemed sufficient so as to adequately describe the nonlinearity of the circuit.

Combining the previous two equations yields

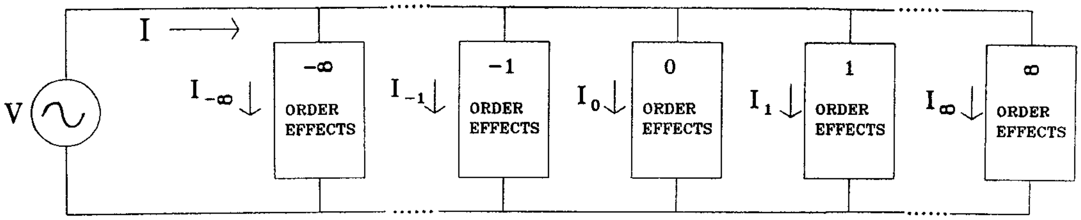


Figure 1: Equivalent Circuit Representation

$$I = \sum_{i=-\infty}^{\infty}\left[K_{i\phi} + \left(\sum_{j=1}^{n} K_{ij}V^{j-1}\right)\frac{\partial^{i} V}{\partial t^{i}}\right] \qquad (3)$$

or in more general terms where $\mathbf{X_{(t)}}$ is the circuit input and $\mathbf{Y_{(t)}}$ is the circuit output,

$$Y_{(t)} = \sum_{i=-\infty}^{\infty}\left[K_{i\phi} + \left(\sum_{j=1}^{n} K_{ij}X_{(t)}{}^{j-1}\right)\frac{\partial X_{(t)}}{\partial t^{i}}\right]. \qquad (4)$$

The procedure for determining the K_{ij} constants is essentially a frequency domain technique requiring as inputs the amplitude and phase for each harmonic output component generated when the circuit is driven by a single frequency, large-signal sinusoidal waveform of sufficient amplitude to drive the circuit in a nonlinear fashion. These harmonic values are obtained for various input signal frequencies.

Assuming that the actual harmonic measurements are a summation of all the differential order effects within a circuit, the harmonic outputs for each individual differential order are obtained by mathematically extracting them from the original harmonic output measurements by the process of fitting, through least-squares regression (3), a polynomial relating the magnitude and phase of each harmonic as a function of the input frequency ω. For example, consider a circuit in which the output contains a DC component plus three harmonics. A total of four polynomials would need to be generated; one describing how the magnitude and phase of the DC component of the output varies as a function of the input signal frequency, a second describing the behavior of the first harmonic output as a function of the input signal frequency, and so on.

This polynomial is of the form

$$Y_{(\omega)} = C_{-\infty}\omega^{-\infty} + \ldots + C_{-1}\omega^{-1} + C_{o} + C_{1}\omega^{1} + \ldots + C_{\infty}\omega^{\infty} \qquad (5)$$

where the C_i constants consist of real and imaginary components.

This polynomial yields the information necessary to determine which differential order "blocks" are needed to represent the circuit by an equivalent circuit of the form shown in Figure 1. Due to the differentiating properties of a single tone sinusoid (the circuit input), the j^{th} derivative of a sinusoid will result in a sinusoid multiplied by both a ω^{j} term and a constant representing the magnitude of the sinusoid. Therefore, the ω terms that are significant in the frequency dependent polynomial indicate which differential order "blocks" are needed in the equivalent circuit.

The polynomials describing the relationship of each harmonic output to the input signal frequency are combined to yield mathematically synthesized harmonic outputs for each differential order represented in the equivalent circuit.

It should be noted that harmonic measurements must be taken at a sufficient number of input signal frequencies so as to adequately characterize the differential orders that are generated within the circuit. For example, a circuit containing an inductor, resistor, and capacitor would contain three different differential orders and therefore harmonic measurements must be taken for at least three different input signal frequencies.

The coefficients describing the output-input characteristic for each differential order are determined by substituting the magnitude and phase of the sinusoidal input signal, and the differential order harmonic outputs "synthetically" generated through the polynomial fitting technique described previously, into eqn. (4), and then equating terms of equal frequency on both sides of the equation. This procedure is performed by a computer algorithm.

The amplitude and frequency validity range of the transfer function is defined by the input signal swing used to generate the output harmonics and the frequency range over which the input signal is varied.

A possible problem in implementing this technique is encountered in obtaining both the magnitude and phase of the harmonic outputs, especially at high frequencies. One method for getting around this problem is that if it is possible to obtain a time domain representation of the output, this data can be passed through a Fast Fourier Transform (FFT) algorithm to yield the harmonic magnitudes and phases. This is the technique that has been used to date. The phase of each harmonic cannot be measured with a standard spectrum analyzer, whereas a FFT of the time domain waveform will generate the desired magnitudes and phases. If this too is not possible, a mathematical model of the circuit can be used to generate the harmonic outputs. The disadvantage in doing this is that the generated transfer function is only as accurate as the mathematical model of the nonlinear circuit. The inherent benefits of obtaining a transfer function representation, however, are still intact. If the circuit being analyzed can be reasonably approximated by an equivalent circuit containing only "zeroeth" differential order effects (no frequency dependency, just resistive effects), then the transfer function coefficients K_{ij} (j=0,n) can be calculated directly by fitting an n^{th} order polynomial to the DC characteristic of the circuit.

Examples

Three examples are given to illustrate the use of this nonlinear circuit transfer function technique. The first compares experimental results to those obtained by the transfer function technique in a diode-resistor circuit where the input is a sinusoidal voltage source and the circuit output is the current flowing through the diode and resistor. This circuit is shown in Figure 2.

A series of DC measurements were used to fit an exponential equation to the **I-V** characteristic of the circuit. This equation was used in a completely independent routine to determine the harmonic response which then defined the transfer function for this circuit (the least-squares fitting routine mentioned previously was not written at the time that this experiment was performed).

The accuracy of the generated transfer function is illustrated in Figures 3 and 4. In these two plots, the transfer function results are compared to measurements made on the actual circuit. All frequencies are normalized to 1KHz. Note that the transfer function is valid only within the input signal voltage range used to generate the harmonic outputs (denoted by the vertical dashed lines in Figure 3).

The second example consists of the nonlinear capacitor-diode circuit shown in Figure 5. The transfer function for this example was obtained from harmonic outputs

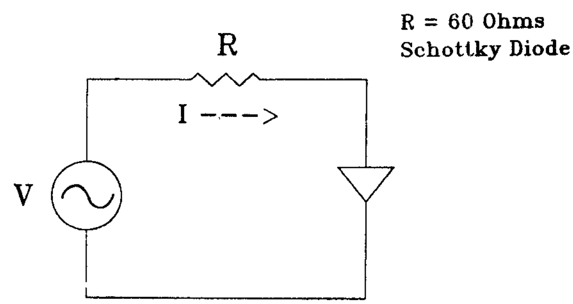


Figure 2: Diode-Resistor Circuit

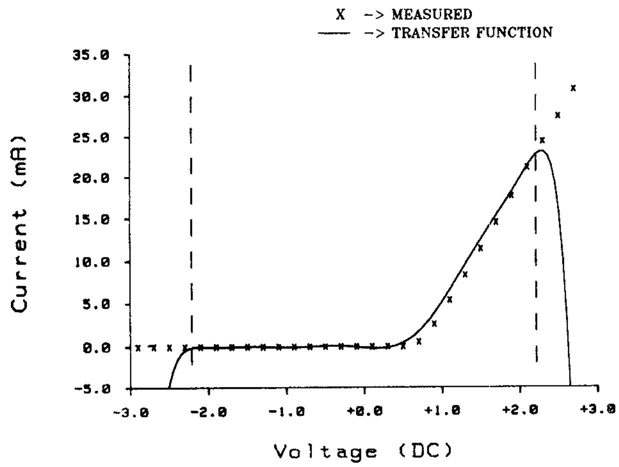


Figure 3: I-V Characteristic Comparisons

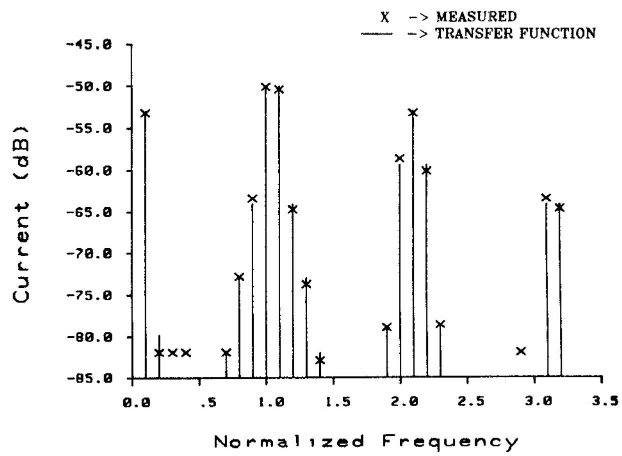
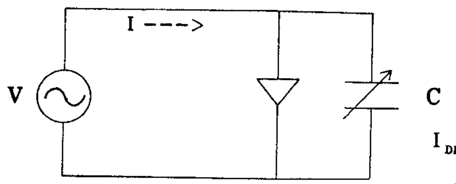


Fig 4: 2-Signal Intermodulation Comparisons

generated by passing time domain data obtained from the theoretical mathematical model of this circuit through a FFT algorithm. This example is used to illustrate some of the benefits of a transfer function as well as the capabilities of the software developed to implement this technique. Figure 6 contains a comparison of theoretical results to those obtained by the transfer function for a 3-signal input.



$$C = \frac{C_o}{(1 + V/\emptyset)^n}$$

$C_o = 50pF$
$\emptyset = 1.5$
$n = .5$

$$I_{DIODE} = I_o(e^{\alpha V} - 1)$$

$I_o = 1x10^{-12}$
$\alpha = 24$

Fig 5: Diode-Nonlinear Capacitor Circuit

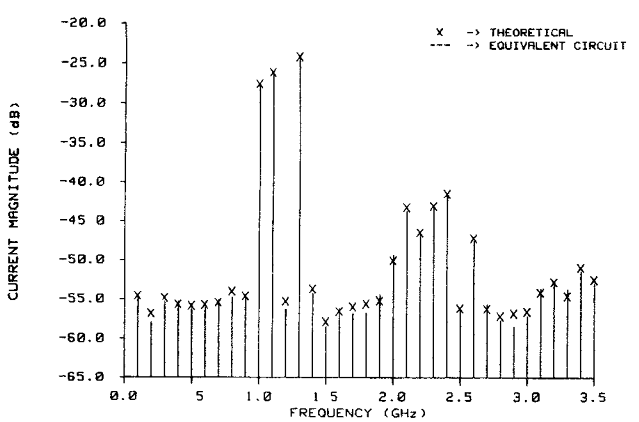


Fig 6: 3-Signal Intermodulation Comparisons

The third example provides an illustration of the possible uses of this technique in determining gain compression, as well as second and third order intermodulation effects, in an amplifier or mixer. A transfer function for a C-band FET was obtained by fitting a polynomial directly to its DC characteristic. This transfer function was used to generate a plot showing the output power of the first, second, and third order components as a function of input power. This plot is shown in Figure 7. Although no data was available to provide a check for these results, they illustrate the potential of using this technique in analyzing power amplifiers and mixers for gain compression, intermodulation distortion, AM to PM conversion, etc.

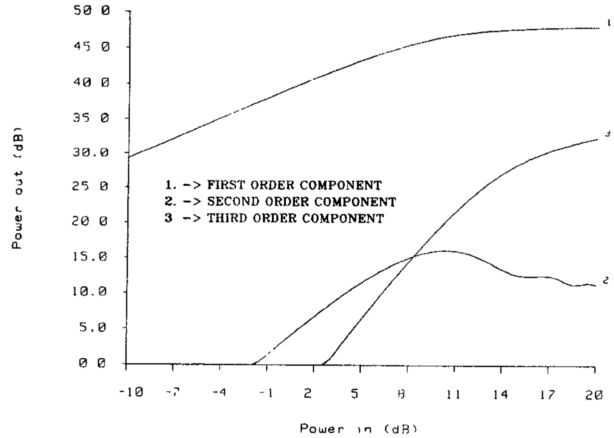


Figure 7: C-Band FET Power Distortion

Conclusion

This paper has presented a frequency domain technique for obtaining a transfer function of a nonlinear circuit. Comparisons of experimental and theoretical results to those results obtained using the technique described in this paper are provided to illustrate the validity and accuracy of this method. Also, an example of the use of this technique in analyzing a microwave device is provided to illustrate the potential uses this transfer function approach has as an aid to the microwave designer.

References

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